The isospin dependence of the nuclear equation of state near the critical point

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Recently, part of our reactions work has concentrated on probing the quantum nature of the liquid-gas phase transition using isotopic yield distributions [1] which may be discussed in terms of a modified Fisher model [2, 3]:

$$Y = y_0 A^{-\tau} e^{-\beta \Delta \mu A}, \qquad (1)$$

where y_0 is a normalization constant, $\tau = 2.3$ is a critical exponent [2], β is the inverse temperature and $\Delta\mu = F((N-Z)/A)$ is the free energy per particle.

Projectiles of 40 A MeV ^{64}Zn, ^{70}Zn and ^{64}Ni beams were used to irradiate ^{58}Ni, ^{64}Ni, ^{112}Sn,^{124}Sn, ^{197}Au, and ^{232}Th targets. Intermediate mass fragments (IMFs) were detected by a detector telescope placed at 20 degrees relative to the beam direction. For each atomic number 6-8 isotopes of very high quality were identified using the $\Delta E - E$ technique. Multiplicities of IMFs were evaluated from moving source fits. The yields of p, d, t, h and α particles, which were identified by the pulse shape discrimination method, were also evaluated by a moving source fit. After correction for accidental contributions, the multiplicities of ^{6}He and ^{8}He were calculated using the source fit parameters obtained for Li isotopes.

According to the Fisher equation given above, we can compare all systems on the same basis by normalizing the yields and factoring out the power law term. For this purpose we have chosen to normalize the yield data for each system to the ^{12}C yield (I = 0) in that system, i.e. we define a ratio:

$$R = \frac{YA^{\tau}}{Y\left(^{12}C\right)12^{\tau}}, \qquad (2)$$

Our data organized by the order parameter m = (N-Z) /A, (the difference in neutron and proton concentration of the fragment). suggest that we are near the critical point for a liquid gas phase

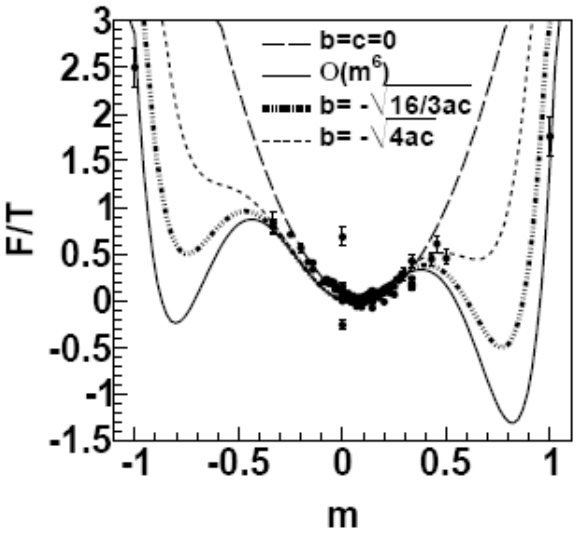

FIG. 1. Free energy versus m for the case ^{64}Ni+^{232}Th. The full line is a free fit based on Landau O(m^6) free energy. The dashed-dotted-dotted-dotted line is obtained imposing in the fit $b = -\sqrt{16/3ac}$ and it is located on a line of first order phase transitions. The short dashed line corresponds to $b = -\sqrt{4ac}$, i.e. superheating. The O(m^2) case, F/T = a(m − m_s)^2,i.e. b = c = 0, ms = 0.1, is given by the long dashed line.

transition (volume and surface terms equal to zero, Coulomb energy contribution not so important compared to the symmetry energy) Fig. 1 shows the quantity F/T= −ln(R)/A versus m = (N-Z)/A. As expected the normalized yield ratios depend strongly on m.

Pursuing the question of phase transitions, in the Laudau approach [1, 4] the ratio of the free energy (per particle) to the temperature is written in terms of an expansion:

$$\frac{F}{T} = \frac{1}{2}am^2 + \frac{1}{4}bm^4 + \frac{1}{6}cm^6 - m\frac{H}{T} \qquad (3)$$

where m is the order parameter, H is its conjugate variable and a−c are fitting parameters.

The use of the Landau approach is for guidance only. A free fit using Eq. (3) is displayed in Fig.1 (full line). The dashed-dotted-dotted-dotted line is obtained imposing in the fit $b = -\sqrt{16/3ac}$ and it is located on a line of first order phase transitions. The short dashed line corresponds to $b = -\sqrt{4ac}$, i.e. superheating. The O(m^2) case, F/T = a(m − m_s)^2,i.e. b = c = 0, ms = 0.1, is given by the long dashed line. Similar to the phase transitions occurring in ^{4}He-^{3}He liquid mixtures, a nucleus, which can undergo a liquid-gas phase transition, should be influenced by the different neutron to proton concentrations. Thus the discontinuity observed in Fig.1 (m = 0) could be a signature for a tricritical point as in the ^{4}He-^{3}He case. We believe that our data, analyzed in terms of the the Landau O (m^6) free energy, suggest such a feature but are not sufficient to clearly demonstrate this.

Once we know the free energy (at least in some cases) we can calculate the NEOS by means of the Fisher model [5]. Since we do not have at present experimental information on the density ρ, temperature T and pressure P of the system we can only estimate the 'reduced pressure' [6]:

$$\frac{P}{\rho T}(m) = \frac{M_0}{M_1}, \qquad (4)$$

where M_i are moments of the mass distribution given by:

$$M_k = \sum_A A^k Y(A,m) = y_0 \sum A^k A^{-\tau} e^{-F/T(m)A}; k = 0,...n \ . \qquad (5)$$

Using the relations above we can calculate the NEOS for the situations illustrated in Fig. 1 but H/T=0 case. The results are displayed in Fig. 2 where the reduced pressure is plotted versus m for vaporization, superheating and first order phase transitions on the tri-critical line. Notice that there is not a large difference between the first two cases, while the last case displays two critical points (a third one is on the negative m axis).

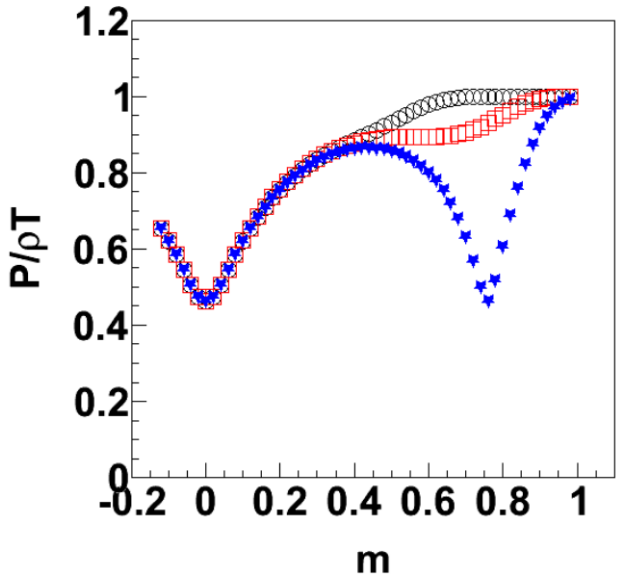

FIG. 2. Reduced pressure versus m of the fragments obtained from the a, b and c parameters fit to the Landau free energy, Eq.(3) for the ^{64}Ni+^{232}Th. The curves correspond to vapor(open circles), superheating(open squares), first order (3 critical line-solid stars), see text.

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